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SETS, SUB-SETS AND OPERATIONS.

BY- FOLEY, JACK L.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE (1) RECOGNIZING SETS AND THEIR MEMBERS, (2) EQUIVALENT SETS AND EQUAL SETS, (3) FINITE AND INFINITE SETS, (4) OPERATIONS WITH SETS, (5) PICTURING SET RELATIONSHIPS, AND (6) SUBSETS AND THEIR COUNT. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

SETS
SUB-SETS
AND
OPERATIONS

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Dr. Jack L. Foley, Director
Elizabeth Basten, Administrative Assistant
Ruth Bower, Assistant Coordinator
Wayne Jacobs, Assistant Coordinator
Gerald Burke, Assistant Coordinator
Leroy B. Smith, Mathematics Coordinator for Palm Beach County

Graduate and Student Assistants

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Scotty Mullinix
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Barbara Miller
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November 1967

For information write: Dr. Jack L. Foley, Director
Building S-503
Sixth Street North
West Palm Beach, Florida 33401

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Recognizing Sets and Their Members

A set can be thought of as a collection of things: a collection of books, numbers, nuts, or people.

A capital letter is used to denote a set and braces { } are used to enclose the members. Below are some examples of sets and set notation.

Example I: Represent the first five whole numbers in set notation (any capital letter can be selected):

$$A = \{0, 1, 2, 3, 4\}$$

Example II: Represent the name of four friends in set notation.-

$$B = \{\text{Jim, Jack, Betty, Tom}\}$$

Example III: Represent the names of three trees in set notation.

$$C = \{\text{Oak, Pine, Cedar}\}$$

The number property of a set is the number of members the set contains. The number property is written in this way. Consider set A in Example I.

N (A) = 5 "read" : The number of members in set A is equal to five.

For sets B and C:

$$N(B) = 4$$

$$N(C) = 3$$

If a set has a number property of zero, it is called an empty set or null set. Braces are used--showing no member inside.
Example: The seasons beginning with the letter P.

$$S = \{\ \}$$

Activities

Represent each collection below in "set notation" and give the number property (remember the braces and choose your capital letter).
Example: The names of the days of the week beginning with an S.

$$W = \{\text{Saturday, Sunday}\}$$

$$N(W) = 2$$

1. The names of the months beginning with a J.

2. All odd numbers between 0 and 12.

3. All even numbers between 9 and 21.

4. All whole numbers that will exactly divide (leave a zero remainder) 24.

5. All whole numbers that 3 will exactly divide between 0 and 30.

6. The names of the snakes in Florida that are poisonous.

7. All single digit whole numbers.

8. The days of the week beginning with a Z.

9. The months of the year beginning with a Q.

10. All even prime numbers.

11. All prime numbers between 1 and 15.

12. All whole numbers between 1 and 35 that are exactly divisible by 2 and 3.

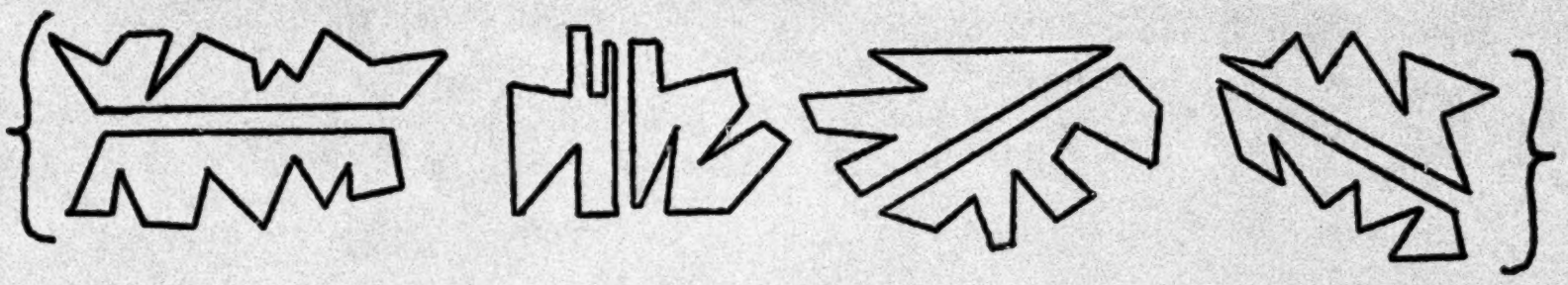
13. All multiples of 3 between 10 and 40.

14. All whole numbers that are greater than zero but less than one.

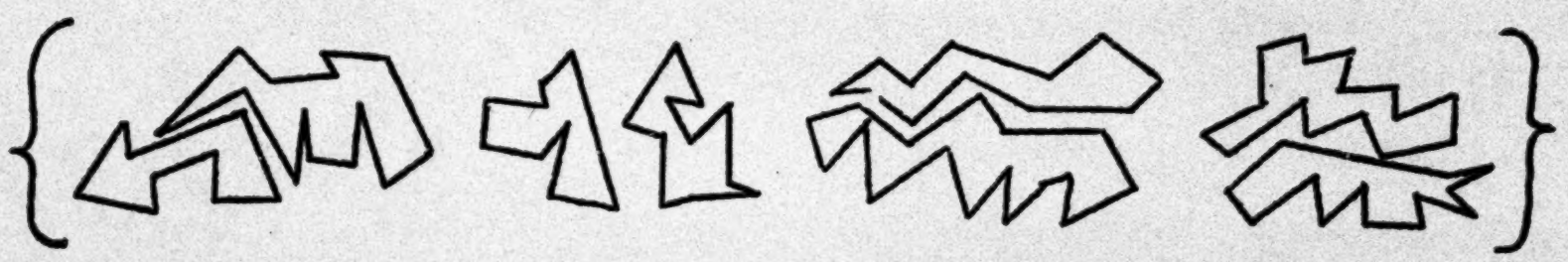
Recognizing members of sets

Enough members of each set on the next page are given to reveal a common characteristic. Some things are shown that do not have this characteristic. Study each and see if you can pick out other members of the set.

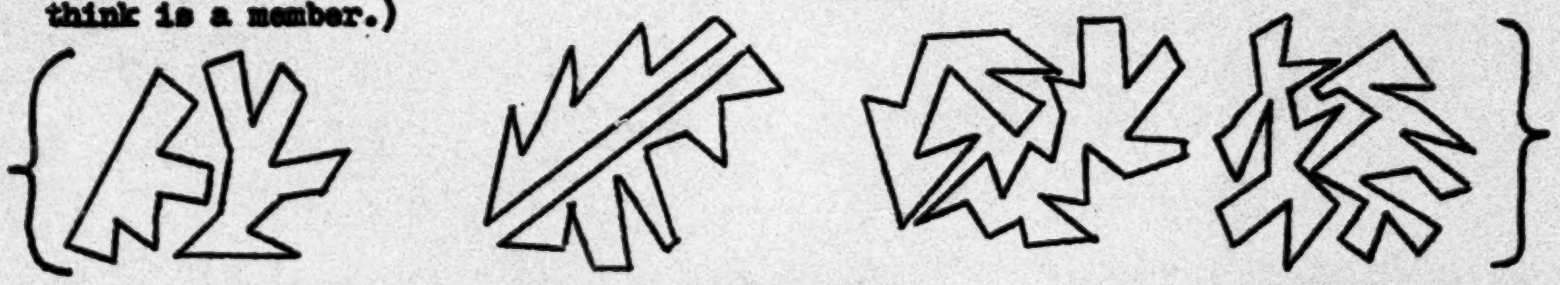
Here are some members of the set of Zig-Zags.



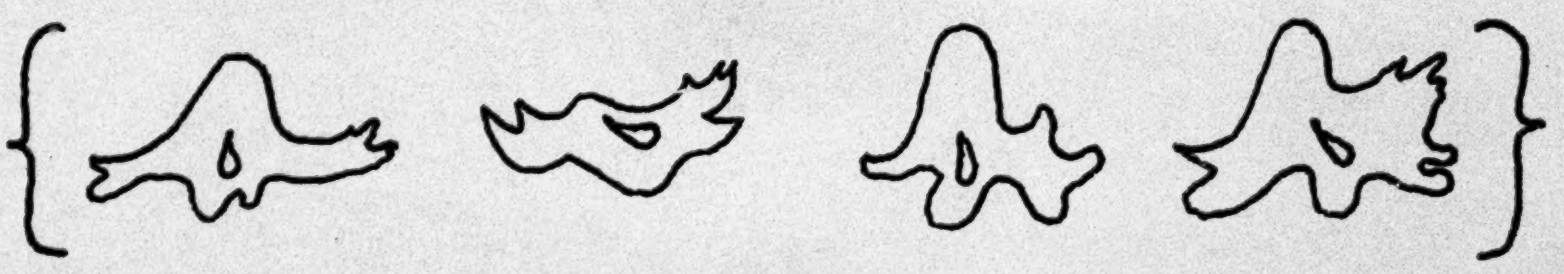
These are not members of the set of Zig-Zags.



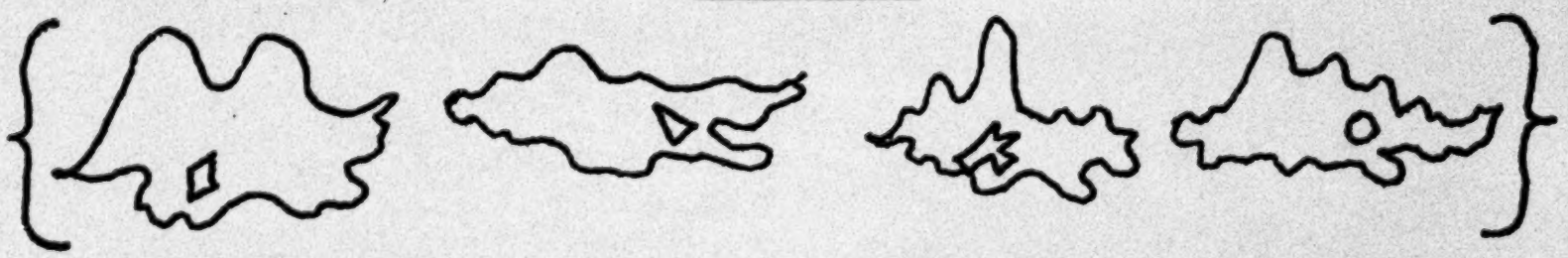
Which of these are members of the set of Zig-Zags? (Circle each that you think is a member.)



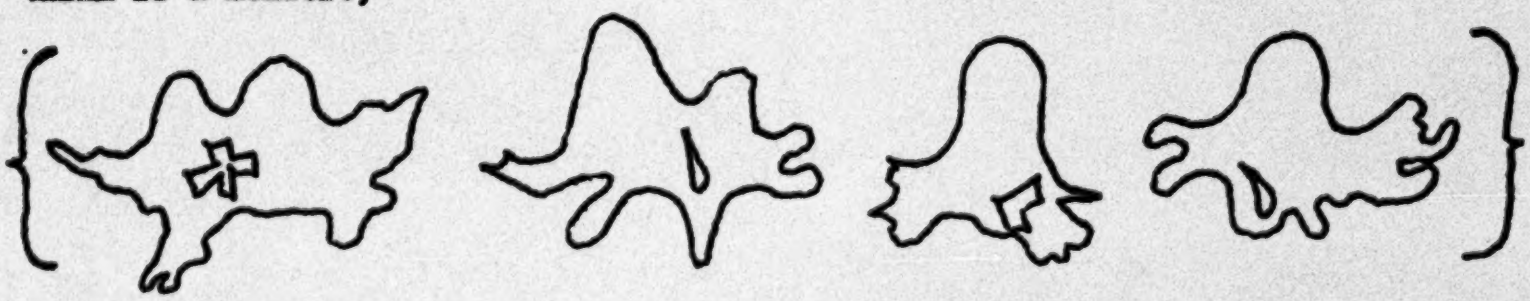
Here are some members of the set of Soggy-Flakes.



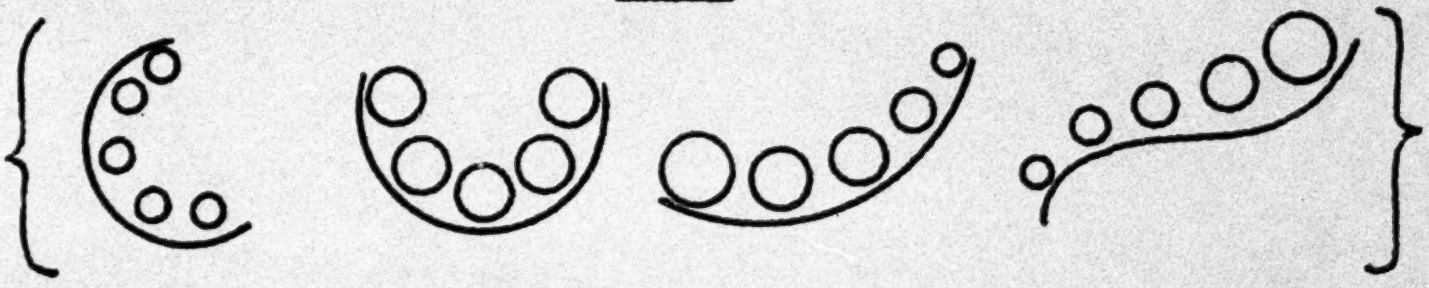
These are not members of the set of Soggy-Flakes.



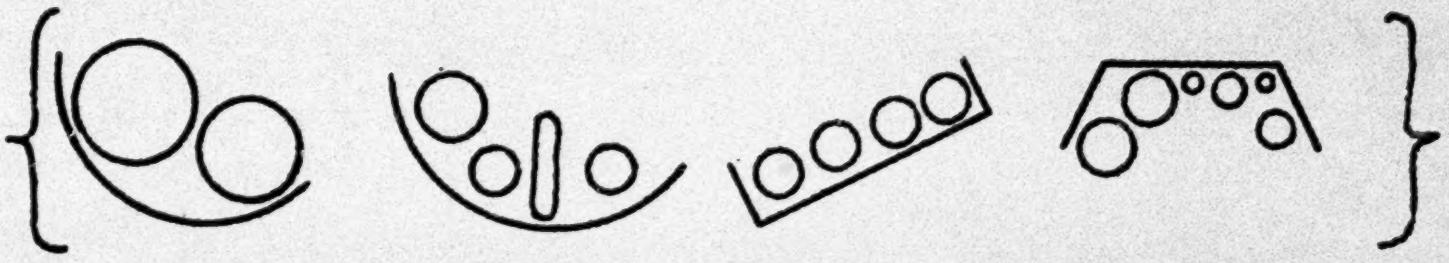
Which of these are members of the set of Soggy-Flakes? (Circle each that you think is a member.)



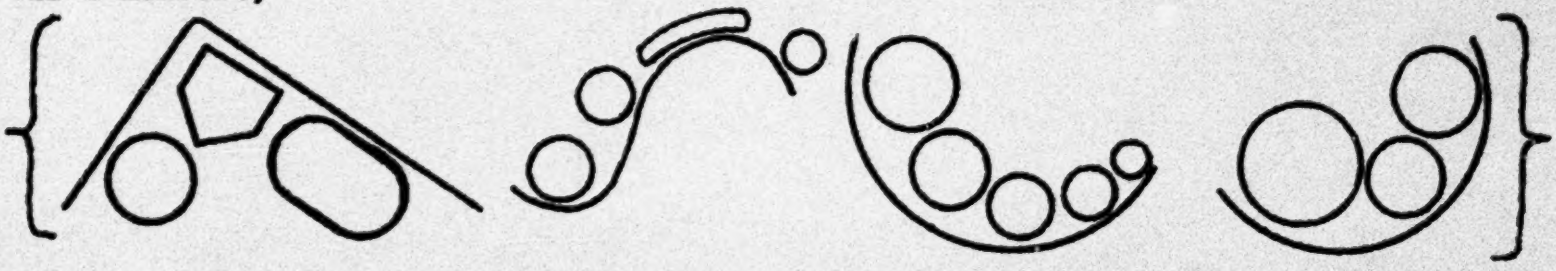
Here are some members of the set of P-Pods.



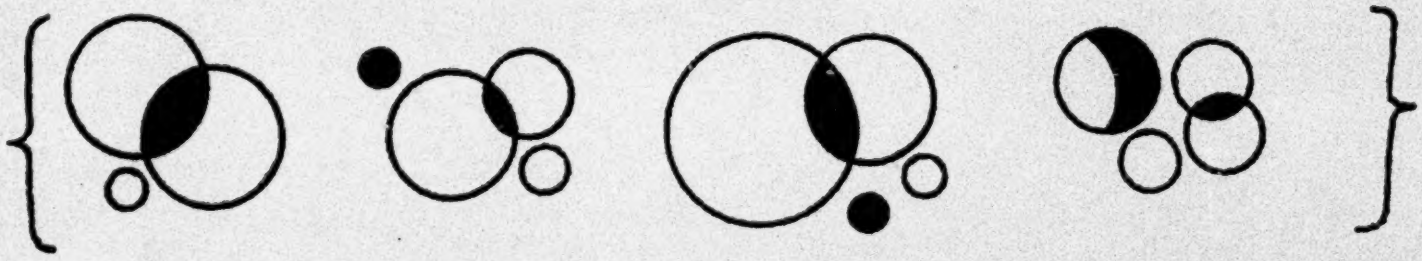
These are not members of the set of P-Pods.



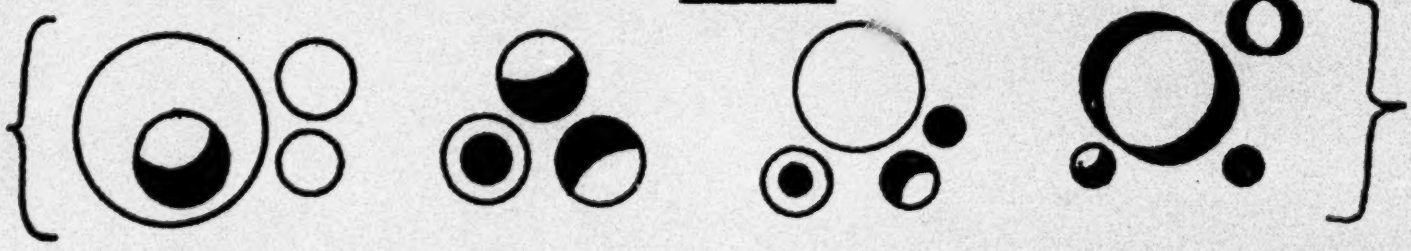
Which of these are members of the set of P-Pods? (Circle each that you think is a member.)



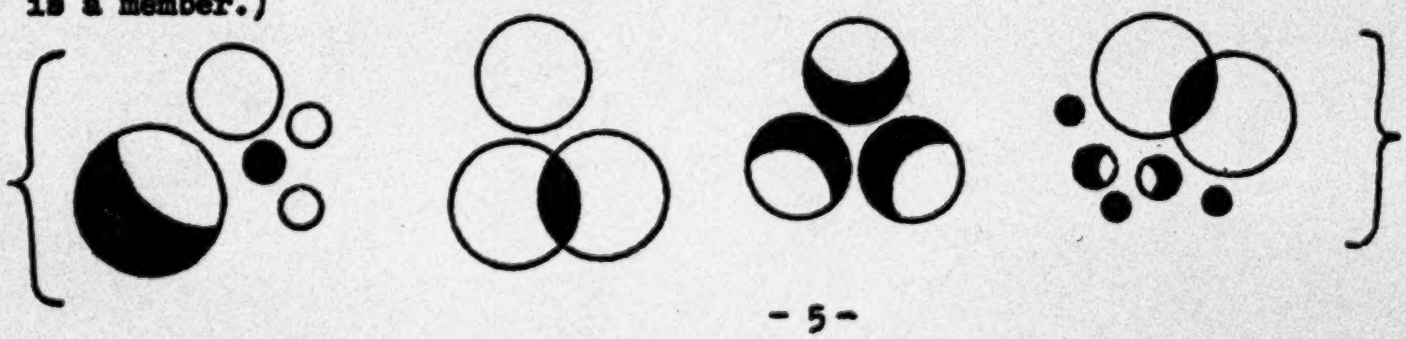
Here are some members of the set of Dit-Dots.



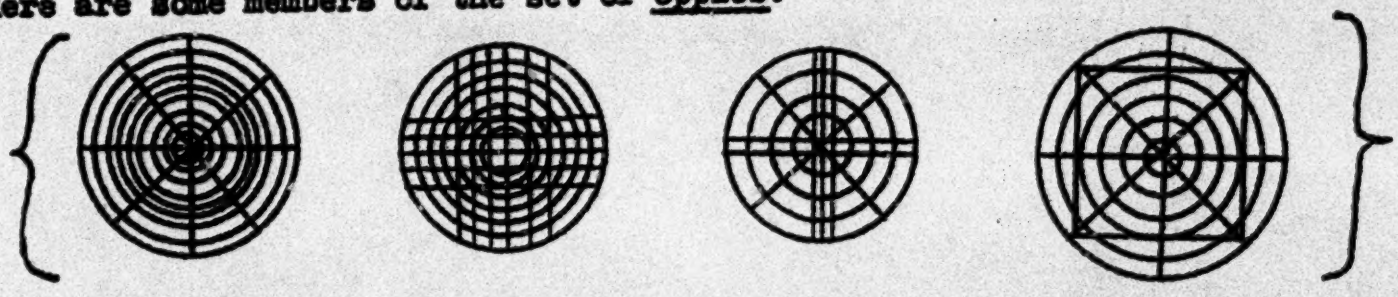
These are not members of the set of Dit-Dots.



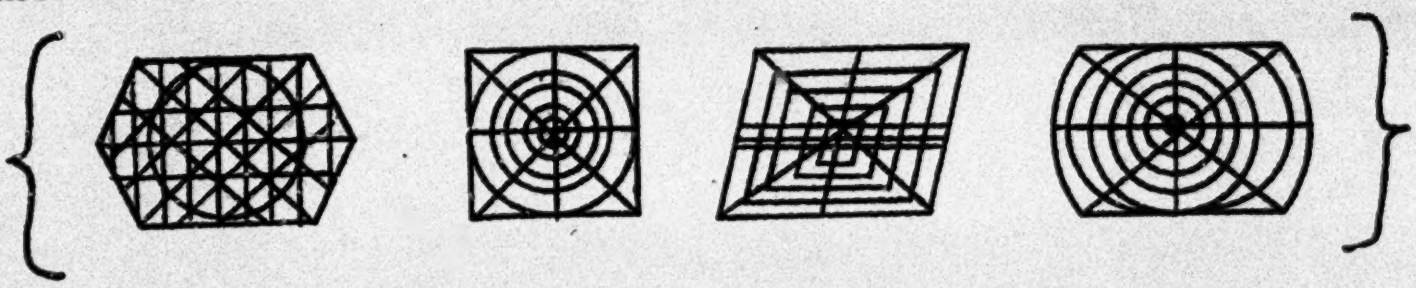
Which of these are members of the set of Dit-Dots? (Circle each that you think is a member.)



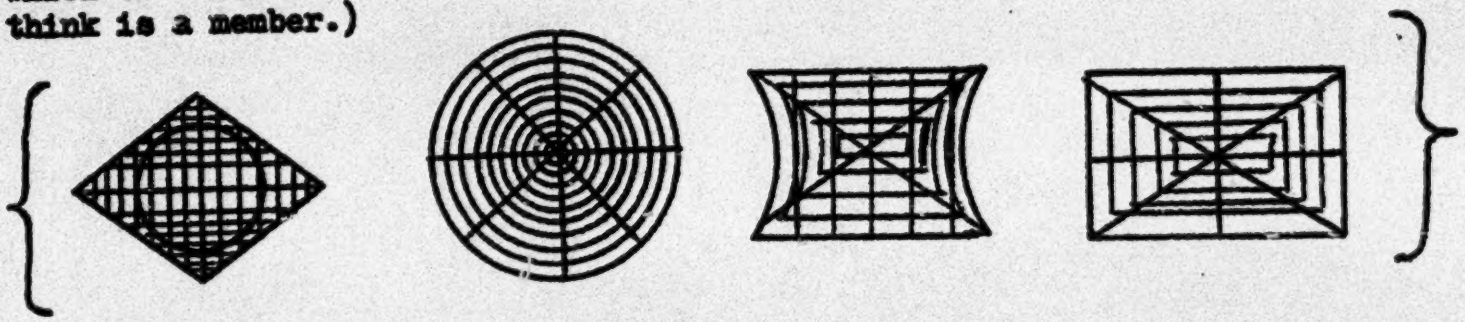
Here are some members of the set of Opples.



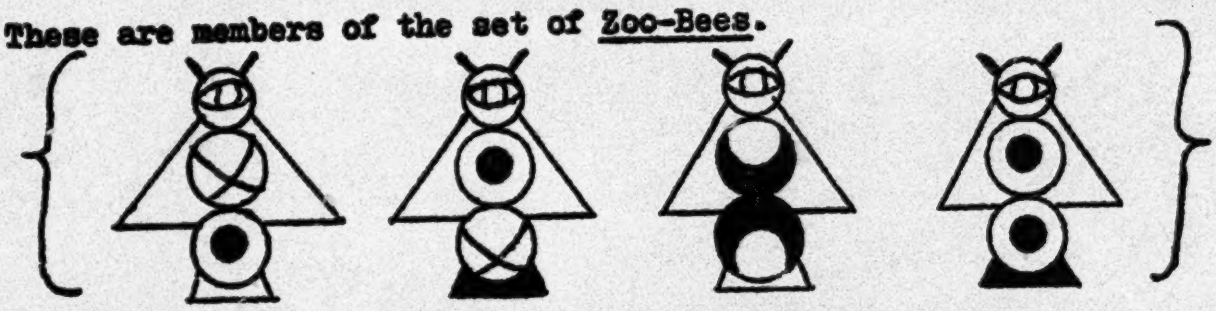
These are not members of the set of Opples.



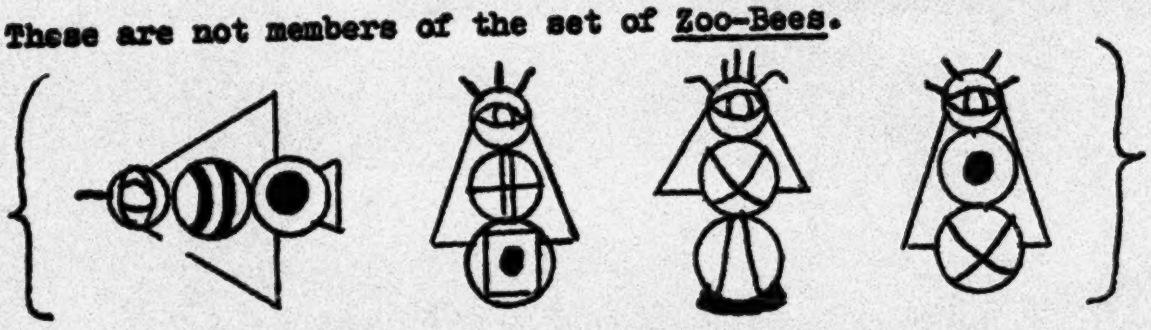
Which of these are members of the set of Opples? (Circle each that you think is a member.)



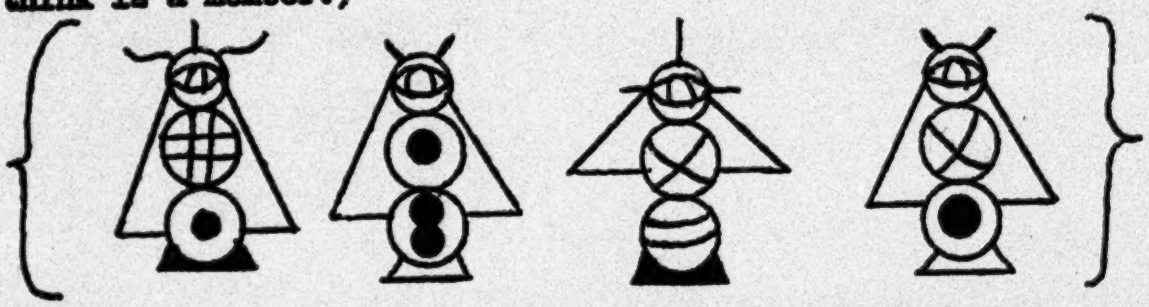
These are members of the set of Zoo-Bees.



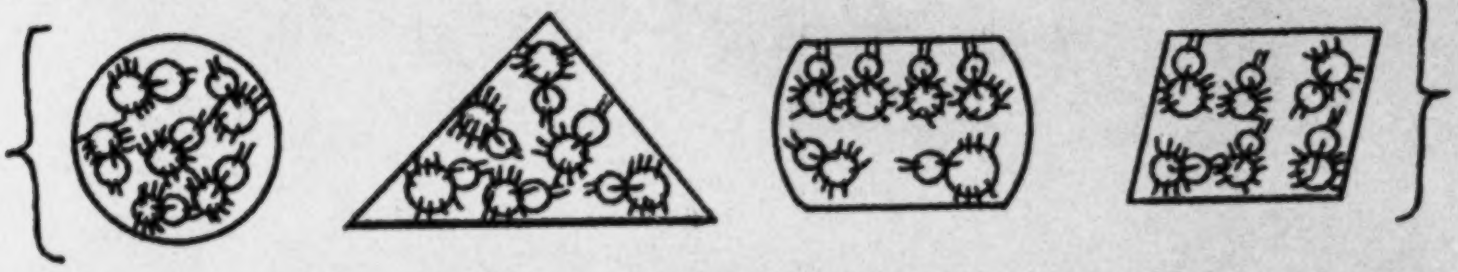
These are not members of the set of Zoo-Bees.



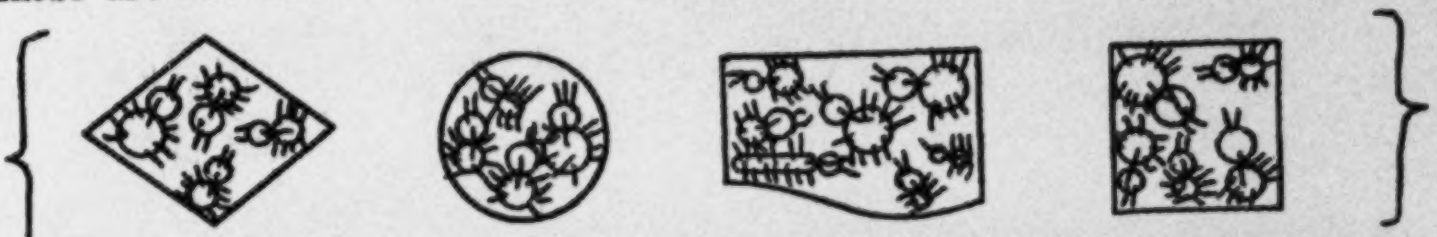
Which of these are members of the set of Zoo-Bees? (Circle each that you think is a member.)



Here are some members of the set of Mini-Mit

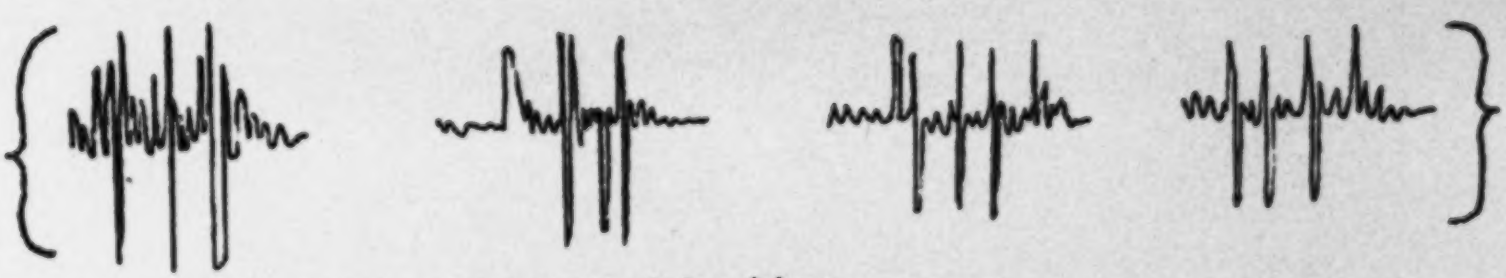


These are not members of the set of Mini-Mites.

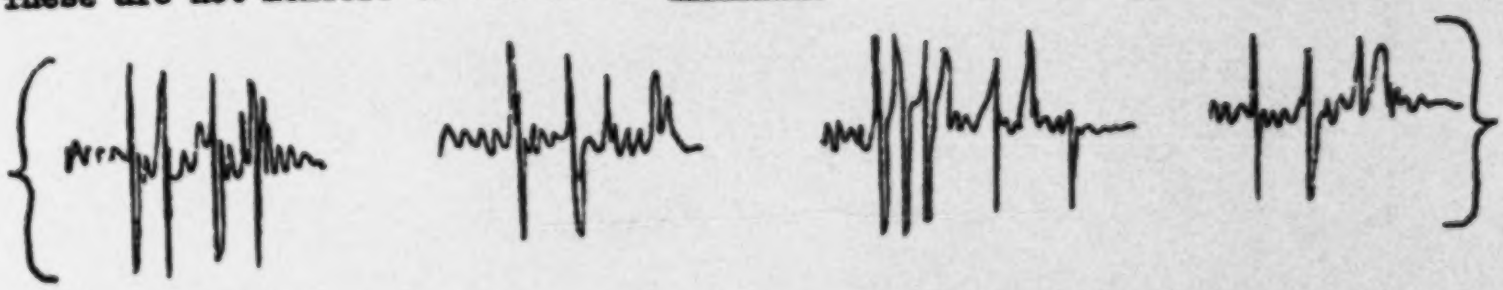


Which of these are members of the set of Mini-Mites? (Circle each that you think is a member.)

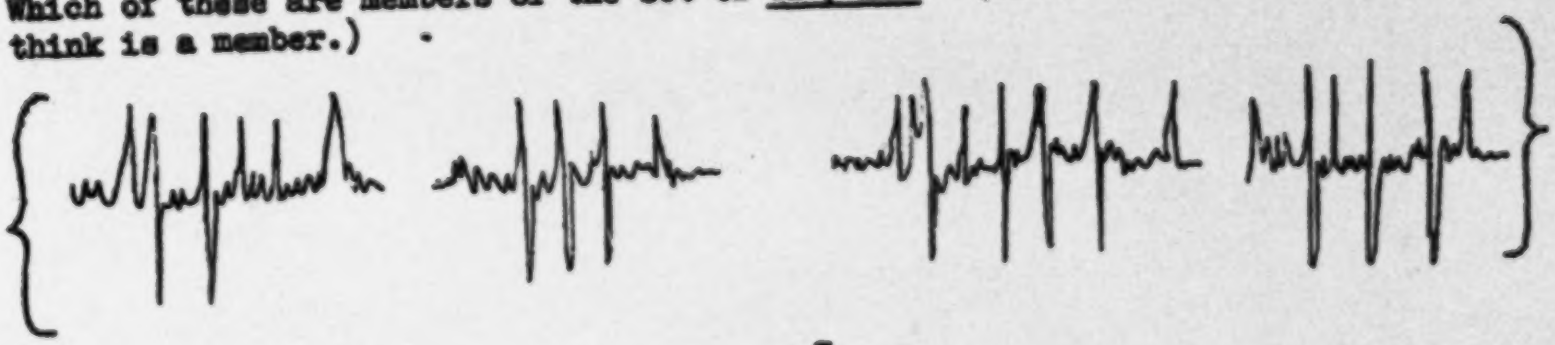
Here are some members of the set of Graphies.



These are not members of the set of Graphies.



Which of these are members of the set of Graphies? (Circle each that you think is a member.)



Comparing and Naming Sets

What do the two sets below have in common?

$$A = \{a, e, i, o, u\}$$

$$B = \{0, 2, 4, 6, 8\}$$

Did you notice? Their number properties are equal:

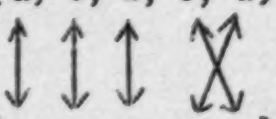
$$N(A) = N(B)$$

$$5 = 5$$

Equivalent Sets: If the number properties of two or more sets are equal, they are called equivalent sets.

Members of equivalent sets can be set up in a one-to-one correspondence or sometimes called a one-to-one matching. This means that each member of a set can be matched with one and only one member of a second set. The idea is shown below with lines drawn to match members.

$$A = \{a, e, i, o, u\}$$



$$B = \{0, 2, 4, 6, 8\}$$

Equal Sets: Two or more sets are equal if they contain, in some order, exactly the same members.

If: $A = \{a, e, i, o, u\}$ and $B = \{e, i, a, o, u\}$

Then: $A = B$

Notice that rearranging members does <u>not</u> make a different set. Also note that, if two sets are equal, they are also equivalent--why? Are equivalent sets always equal? Certainly not.

Activities

Circle "equivalent," if the two sets are equivalent, and "equal," if they are equal. Circle "neither" if the sets are neither equivalent nor equal (remember that equal sets are always equivalent).

The first is an illustration.

1. $\{a, b, c, d\}$ and $\{0, 2, 4, 6\}$

a. (equivalent) b. equal c. neither

2. $\{a, b, c\}$ and $\{0, 2, 4, 6\}$

a. equivalent b. equal c. neither

3. $\{1, 3, 5, 7\}$ and $\{5, 7, 1, 3\}$

a. equivalent b. equal c. neither

4. $\{\triangle, \bigstar, \bigcirc, \square\}$ and $\{3, 4, 5, 1\}$

a. equivalent b. equal c. neither

5. $\{q, r, s, t\}$ and $\{r, s, q\}$

a. equivalent b. equal c. neither

6. {John, Jim, Jack} and {Jack , Jim, Jane }

a. equivalent b. equal c. neither

7. {heads in your room} and {people in your room }

a. equivalent b. equal c. neither

8. {boys in your room} and {girls in your room }

a. equivalent b. equal c. neither

9. {heads in your room} and {feet in your room }

a. equivalent b. equal c. neither

10. {teachers in your room} and {students in your room }

a. equivalent b. equal c. neither

11. Review: Give the number property of each set (a - e). Follow the example--use the same form.

Example: $A = \{a, b, c, d\}$; $N(A) = 4$

a. $B = \{1\}$;

b. $S = \{\ \}$;

c. $C = \{2, 4, 6, 8, 10, 12\}$;

d. $C = \{0\}$;

e. $G = \{\triangle, \square, 5, e, \pi\}$

Finite Sets: Sets with "whole number" properties are called finite sets.

Another way to state this is that: A set is finite if a whole number describes the number of elements. Example of finite sets are:

a) The set of girls in your school.
b) The set of even numbers between 5 and 15.
c) The set of cities in Florida.

Infinite Sets: Sets that do not have whole number properties.

Examples of infinite sets are:

a) The set of points on a line.
b) The set of whole numbers.
c) The set of even numbers.

With infinite sets of numbers, a few are listed and, usually, three dots are used to show they continue on indefinitely. The whole numbers are often represented as:

$$W = \{0, 1, 2, 3, \ldots\}$$

Sometimes dots are used to show that some members are left out, but if this is the case, a last member is listed.

For example: The set of months of the year:

$$M = \{ \text{January, February, . . . , December} \}$$

The example above, although dots are used, is a finite set. That is:

$$N(M) = 12$$

Activities

Each set below is either finite or infinite. Write in your choice.

_____ 1. { All odd numbers }

_____ 2. { Population of Florida }

_____ 3. { Stars in the universe }

_____ 4. { All even numbers }

_____ 5. { Grains of sand on Palm Beach}

_____ 6. { Drops of water in the Atlantic Ocean }

_____ 7. { Whole numbers exactly divisible by 3 }

_____ 8. {In years--the age of the earth}

_____ 9. {Points on a line }

_____ 10. {Lines in a plane}

Operations With Sets

In general, an operation is a procedure or way of doing "something." Addition is an operation with numbers--it is a way or a procedure of taking addends and finding a sum. Actually it is a "fast" way to count.

The two operations with sets examined in this work will involve "listing." The operations are:

Operation	Symbol Used
1. Union	" $\cup$ "
2. Intersection	" $\cap$ "

First, the "union" of sets will be defined and illustrated:

Union: For any two sets A and B, "list" all members that are in set A or set B.

Notice that by listing all members that are in either set, we get a third set. A member can be "listed" only once.

The first example will illustrate this idea.

Set A = {Planning Committee for the class dance}

= {Joe, Sue, Wayne, Beth, Dale}

Set B = {Planning Committee for class field trips }

= {Gerald, Ruth, Wayne, Jean, Babs}

A "Union" B would be to list all members, without repetition, that are in set A or set B.

A∪B ={ Joe, Sue, Wayne, Beth, Dale, Gerald, Ruth, Jean, Babs }

Notice that "Wayne" is not listed two times. If these two committees met, certainly "two" Waynes would not appear. It is the same Wayne in set A as in set B. If there were two different students named Wayne--something would have to be done to show it is two different people.

Intersection; For any two sets A and B, "list" all members that are in both set A and set B.

For Set A ={ Joe, Sue, Wayne, Beth, Dale }

Set B ={ Gerald, Ruth, Wayne, Jean, Babs}

A "intersection" B ={ members of both sets}

A ∩ B = { Wayne }

Wayne is the only member that is on both committees.

Two more examples are given of the "union" and intersection of sets:

I. $A = \{1, 2, 3, 4, 5, 6\}$; $B = \{4, 5, 6, 7, 8, 9\}$

(union) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(intersection) $A \cap B = \{4, 5, 6\}$

II. $A = \{a, b, c\}$; $B = \{x, y, z\}$

$A \cup B = \{a, b, c, x, y, z\}$

$A \cap B = \{\quad\}$

In example II, sets A and B are <u>disjoint sets</u>. The intersection of <u>disjoints sets</u> gives an <u>empty set</u>.

Activities

Show the indicated set formed by the indicated operation.

1. $A = \{a, r, g, h\}$; $B = \{m, d, t, i\}$

$A \cup B = \{\quad\}$

$A \cap B = \{\quad\}$

2. $A = \{ r, e, y \}$; $B = \{ s, u, r, l \}$

$A \cup B = \{$ $\}$

$A \cap B = \{$ $\}$

3. $A = \{ 1, 2, 3 \}$; $B = \{ a, 2, b \}$

$A \cup B = \{$ $\}$

$A \cap B = \{$ $\}$

4. $A = \{$ all students in your class $\}$

$B = \{$ all students in your school $\}$

$A \cup B = \{$ $\}$

$A \cap B = \{$ $\}$

5. $A = \{ \}$; $B = \{ 1, 2, 3 \}$

$A \cup B = \{$ $\}$

$A \cap B = \{$ $\}$

6. Using: $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$C = \{5, 6, 7, 8, 9\}$

$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Find (supply the braces):

a.) $A \cup B =$

b) $A \cup C =$

c) $B \cap A =$

d) $C \cap D =$

e) $D \cup C =$

f) $C \cup A =$

g) $B \cup D =$

h) $D \cup B =$

Follow the example below to complete the problem. Find:

$$(A \cup B) \cap C$$

First: $A \cup B = \{1, 2, 3, 4, 5, 6\}$

And $C = \{5, 6, 7, 8, 9\}$

Then $(A \cup B) \cap C = \{5, 6\}$

i) $(A \cup B) \cap D =$

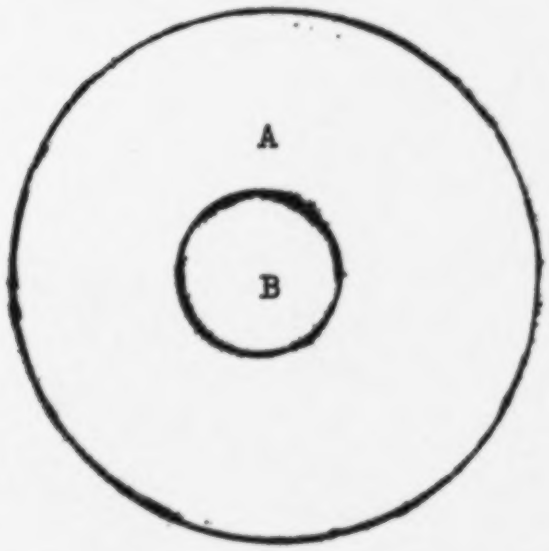
j) $(A \cap B) \cap D =$

Picturing Set Relationships

Circles, called Venn diagrams, are often used to show how sets are related. The members are enclosed by the circles. Below Venn diagrams are used.

I. A = { Students in your school }

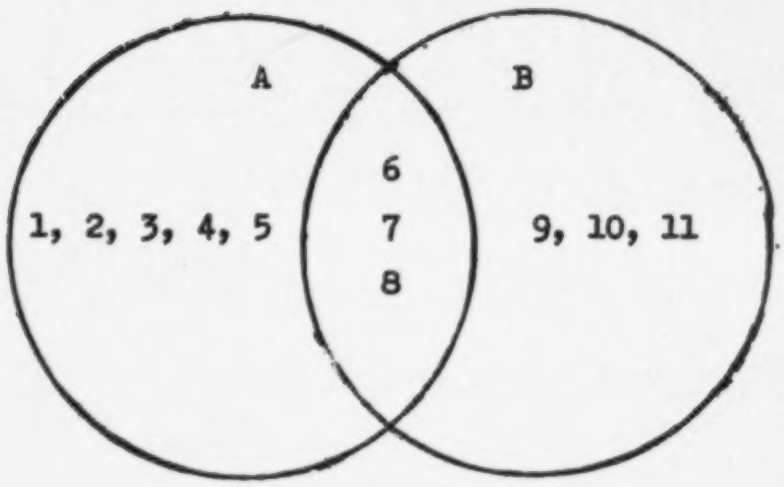
B = { Students in your class }



a) Are all members of B also members of A?

b) Are all members of A also members of B?

II. A = {1, 2, 3, 4, 5, 6, 7, 8}; Set B = { 6, 7, 8, 9, 10, 11 }



a) List the members of set A that are members of set B.

{ }

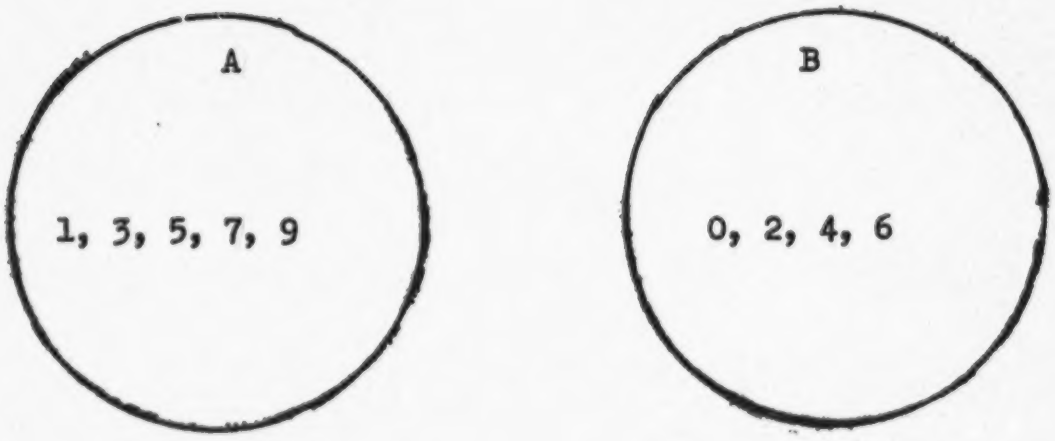
b) List members of set A that are <u>not</u> members of B.

{ }

c) List members of B that are not members of A.

{ }

III. Set A = {1, 3, 5, 7, 9}; Set B = {0, 2, 4, 6}



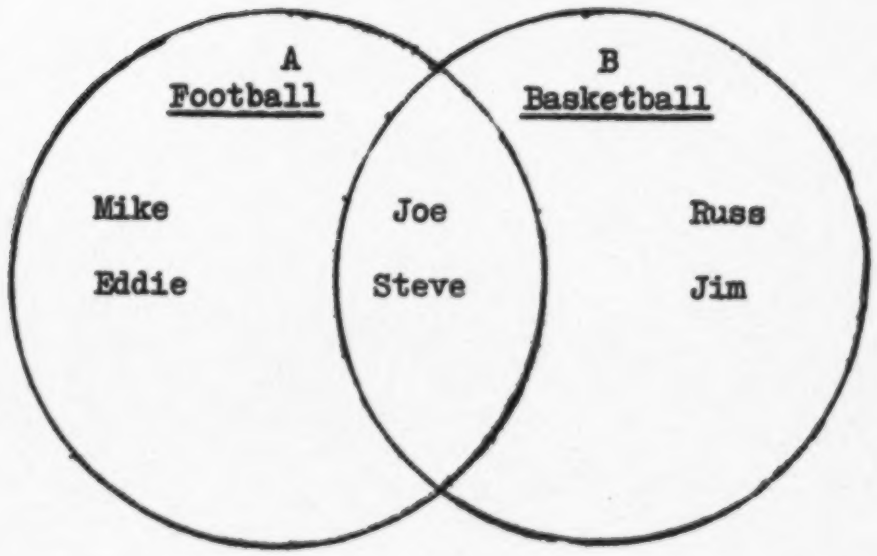
These are disjoint sets:

$A \cap B = \{\quad\}$

Activities

Complete the following problems:

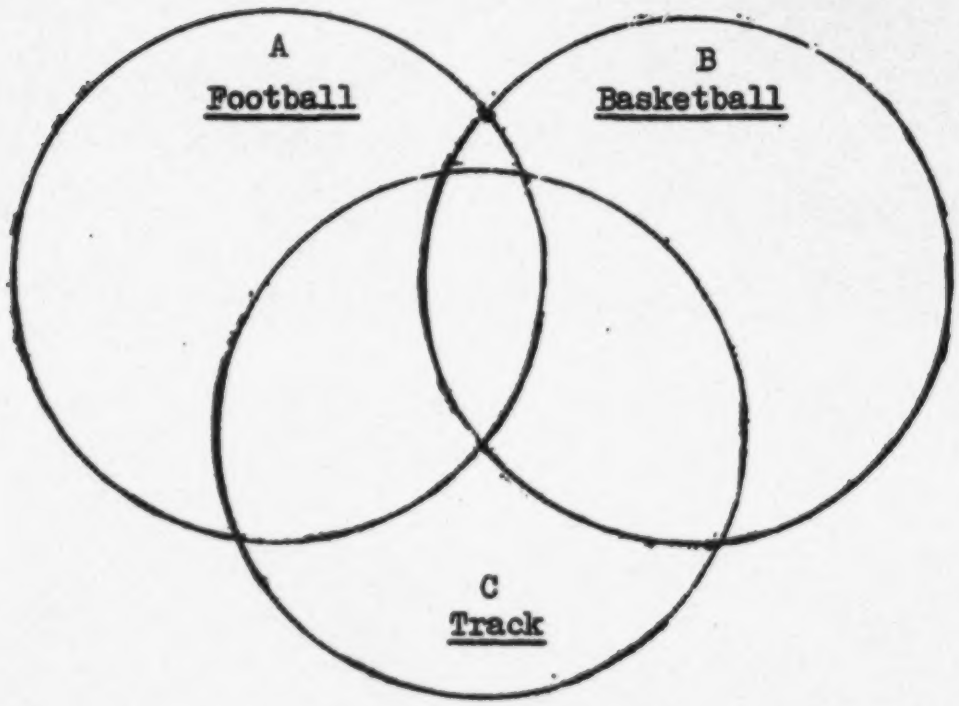
1. The diagram shows the students in a class that are on the football team, the basketball, or on both.



A ∪ B =

A ∩ B =

2. The drawing below shows class students participating in (one, two, or maybe all three) track, football, and basketball.



a) Use vertical lines to shade $A \cap B$.

b) Use horizontal lines to shade $A \cap C$.

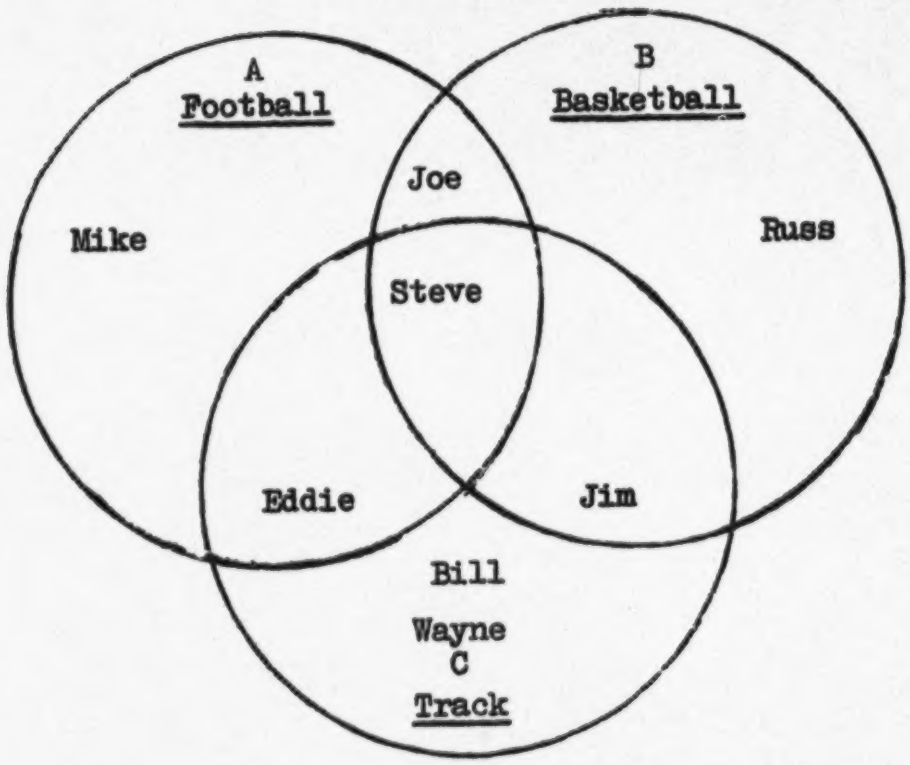
c) Use diagnol lines to shade $B \cap C$.

Is there a shaded area with all three type lines in it?
Is this?

$A \cap B \cap C$

Does this show some students on <u>all</u> three teams?

3. The names are given in the Venn diagrams below. This may assist you in checking your answers to problem 2.



a) List who plays football.

A = { }

b) List who plays basketball.

B = { }

c) List who is on the track team.

C = { }

d) List who plays both football <u>and</u> basketball.

$A \cap B = \{$ $\}$

e) List who plays football and is also on the track team.

$A \cap C = \{$ $\}$

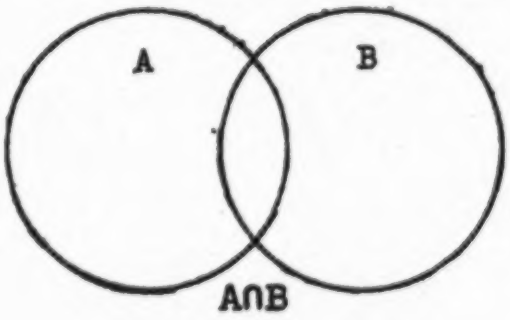
f) List who plays basketball <u>and</u> is on the track team.

$B \cap C = \{$ $\}$

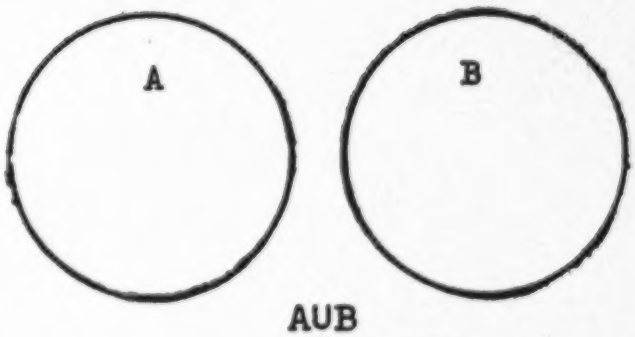
j) List the students that are on all three teams.

$A \cap B \cap C = \{$ $\}$

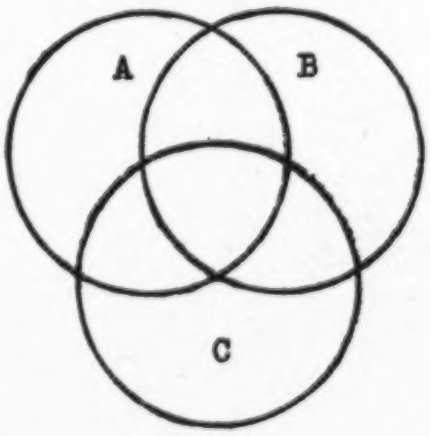
4. Shade the correct region.



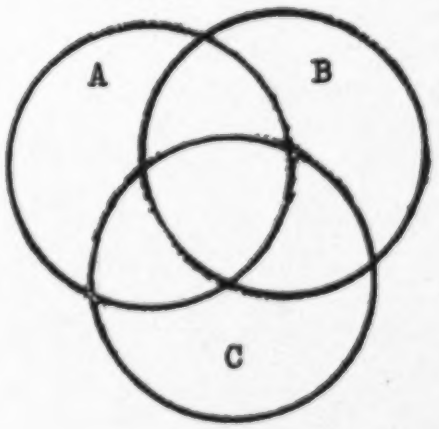
$A \cap B$



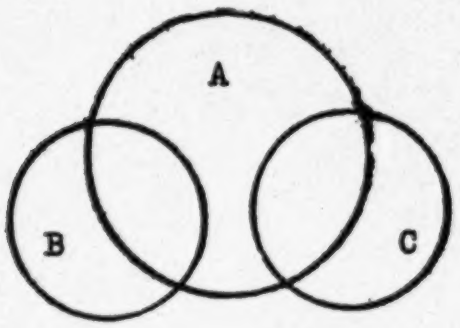
$A \cup B$



$A \cup B$

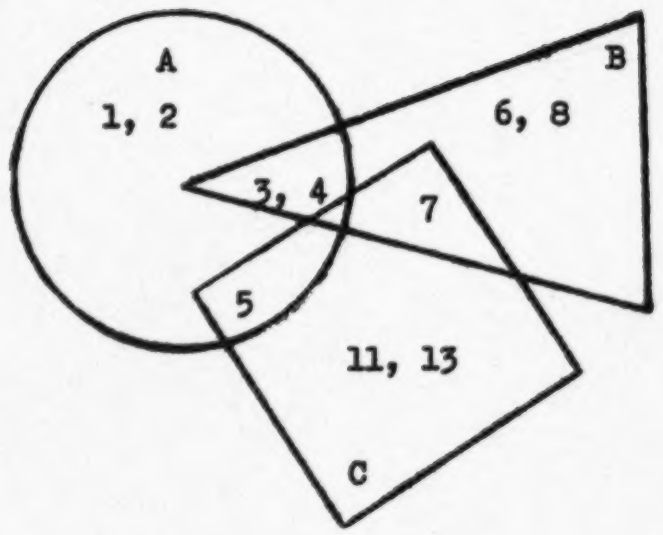


$B \cap C$



$B \cap A$ and $A \cap C$

5. From the diagram below, fill in the members of each set.

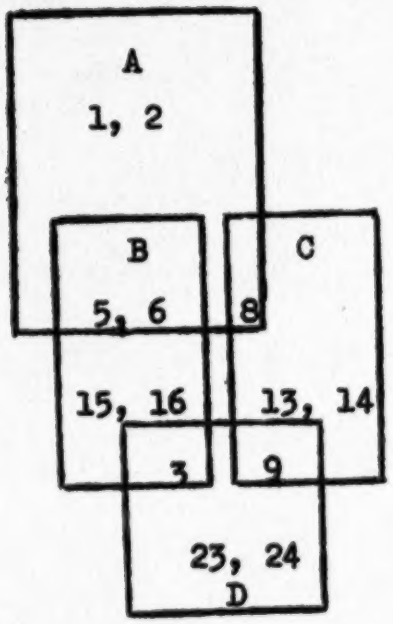


A = { }

B = { }

C = { }

6. From the diagram, complete the problems.



A = { }

B = { }

C = { }

D = { }

a) B∩D = { }

b) C∩B = { }

c) A∩C = { }

d) A∩D = { }

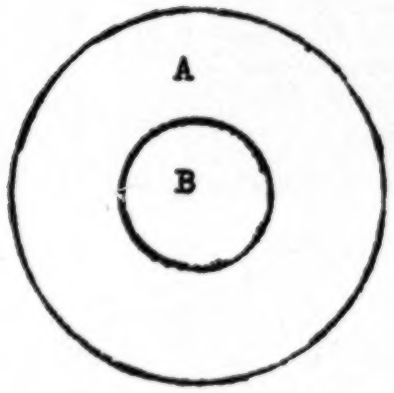
e) A∪B = { }

f) B∪C = { }

g) A∪D = { }

Subsets

Pictured below are two sets. Notice that set B "is contained in" set A.



This means that every member of set B is also a member of set A.

Set B is called a subset of set A. Below is an example.

A = {letters of the alphabet}

B = {vowels}

A symbol is used to show this relationship.

Symbol	Meaning
$\subset$	"is contained in"

then mathematically:

set B "is contained in" set A

or simply:

B C A

Activities

Below are some examples of subsets. State the original set. The first is an example.

Subset	Set
1. {Tuesday, Thursday}	_____
2. {Alaska, Alabama, Arizona}	_____
3. {January, February, March}	_____
4. {0, 2, 4, 6, 8}	_____
5. {1, 3, 5, 7, 9}	_____
6. {students taking math}	_____

7. In the blanks on the next page, see if you can supply a meaningful set, subset, and one member of the subset. One blank or more is filled out.

	Set	Subset	One Member
a)	Money	Coins	_____
b)	Family	Sons	_____
c)	Army	_____	_____
d)	Students	Math Students	_____
e)	Whole numbers	Odd numbers	_____
f)	_____	_____	Planet Earth
g)	_____	Teachers	_____
h)	Animals	_____	_____
i)	Plants	_____	_____
j)	_____	_____	_____
k)	Politicians	_____	_____

Counting Subsets

Each set can be broken down into a definite number of different subsets. The empty set is a subset of every set. Observe the different subsets listed for the set below.

$$A = \{a, b\}$$

Different Subsets

1. $\{\ \ \}$
2. $\{a\}$
3. $\{b\}$
4. $\{a, b\}$

A set having 2 members can be broken into 4 different subsets. Remember--rearranging members does not make a different set.

If a set has 3 members (number property of 3 or N (set) = 3) then it has 8 different subsets.

Complete the list below.

$$S = \{*, \square, \triangle\}$$

Subsets

(The first two are the empty set and the set itself.)

1. $\{\ \ \}$
2. $\{*, \square, \triangle\}$
3. $\{*, \square\}$
4. $\{\ \ \}$
5. $\{\ \ \}$
6. $\{\ \ \}$
7. $\{\ \ \}$
8. $\{\ \ \}$

A pattern showing the number of different subsets a set can be broken into is shown below. The number of different subsets depends on the number property (number of members) of the original set.

	Number Property (Number of members)	Number of Different Subsets
If:	N (S) = 0	1
	N (S) = 1	2 = 2
	N (S) = 2	4 = 2 X 2
	N (S) = 3	8 = 2 X 2 X 2
	N (S) = 4	16 = 2 X 2 X 2 X 2
	N (S) = 5	___ = __X__X__X__X__

Can you describe the pattern?
(Clue--if the set has 4 elements, 2 is listed as a factor 4 times.)

Activities

1. Using the set below, list all different subsets--you should get 16.

$$S = \{ a, b, c, d \}$$

2. A set having 6 members will have how many different subsets.

Subsets are listed according to their number property. The number of subsets with the same number property is recorded. These numbers form an interesting pattern.

The subsets arranged by the number of elements they contain form a triangle like this.

```
                1
             1     1
          1     2     1
       1     3     3     1
    1     4     6     4     1
 1     5    10    10     5     1
1     6    15    20    15     6     1
```

Have you seen this before? Can you continue it?

In a set with 7 elements, how many subsets would you have that have 3 elements?

Number Property	Sample Set	Subsets	Total Subsets
0	{ }	1 { }	1
1	{a}	1 1 { } and {a}	2
2	{a,b}	1 2 1 { }; {a} {b}; {a,b}	4
3	{a,b,c}	1 3 3 1 { }; {a} {b} {c}; {a,b} {a,c} {b,c}; {a,b,c}	8
4	{a,b,c,d }	1 4 6 4 1 { }; {a} {b} {c} {d}; {a,b} {a,c} {a,d} {b,c} {b,d} {c,d}; {a,b,c} {a,c,d} {b,c,d} {a,b,d}; {a,b,c,d }	16
5	{a,b,c,d,e}	1 Empty set; 5 Sets with 1 element; 10 Sets with 2 elements; 10 Sets with 3 elements; 5 Sets with 4 elements; 1 Sets with 5 elements	32
6	{a.b,c,d,e,f}	1 6 15 20 15 6 1 0ε 1ε 2ε 3ε 4ε 5ε 6ε	64

CAN YOU SEE THE PATTERN?